CALCULATION

Subject	Coronavirus in UK	By	J. Bush	Date	10/04/2020

BACKGROUND

The World is caught in the Coronavirus pandemic, with governments imposing lockdowns to slow the infection.
In the UK it is very unclear how widespread the infection is, with only 173,000 tests (as of 3rd April) for a population of 65 million.
Moreover the true morbidity of the disease is not known, but is speculated to be much higher than the 0.1% morbidity for influenza.

The purpose of this calculation is to use the progression of the deaths due to the infection to calculate the morbidity and extent of the infection in the United Kingdom.

ASSUMPTIONS

1. The infection rate constant has been constant in the UK up to 3rd April 2020.
2. The mortality has been constant in the UK up to the 3rd April 2020.
3. The infection is deemed to have started on 14th January 2020 c.f. reports of skiers returning home with Coronavirus symptomes at this time.
4. Death occurs after a fixed time following infection.

METHOD

An equation is derived for the infection rate constant from observations of the deaths due to the infection.
This equation is used to calculate the infection rate constant from observations of the deaths.
An equation is derived for the morbidity and number of infections based on the infection rate constant.
The mortality and number of infections are calculated from the infection rate constant.

RESULTS

Mortality =	0.29	%	See sheet 3.
People infected as of 3rd April 2020 =	24.4	Million People	See sheet 3.

CONCLUSIONS

The results show that around 36% of the UK population had the infection or have had it, as of 3rd April 2020.

If the results are true then there is no point in prolonging the lockdown beyond the end of this month, as most people will become infected during the next few weeks even if the lockdown succeeds in slowing the rate of infection.

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Calculation of the infection rate constant.

The number of infections is given by equation 4 and is characterised by the infection rate constant k.
Equation 4 is derived on sheet 4.

$$N_I = e^{kt}$$ Equation 4

Where:
t = Time in days.
N_I = Number of infected people at time t.
k = Infection rate constant.

The constant k can be calculated from observations of the deaths caused by the virus, using equations 10 and 11:
Equations 10 and 11 are derived on sheet 5.

$$k = \frac{1}{\Delta t} \cdot \ln\left(\frac{N_D\,(t + \Delta t)}{N_D\,(t)}\right)$$ Equation 10

$$k = \ln\left(\frac{N_D\,(t + 1)}{N_D\,(t)}\right)$$ Equation 11

Where:
$N_D(t)$ = Number of people killed by coronavirus at time t = t.
Δt = The time in days between successive observations of the total number of deaths due to the virus.
$N_D(t+\Delta t)$ = The number of deaths due to the virus at time t = t + Δt.

Using equations 10 and 11 the constant k is calculated from the deaths due to the virus observed in Britain and Germany, as follows.

Date	Deaths in UK*	k (eq. 10) day^{-1}	Δt (eq. 11) days	k (eq. 11) day^{-1}	Deaths in Germany*	k (eq. 10) day^{-1}	Δt (eq. 11) days	k (eq. 11) day^{-1}
22/03/2020	281				94			
23/03/2020	335	0.176	1	0.176	118	0.227	1	0.227
24/03/2020	422	0.231	2	0.203	149	0.233	2	0.230
25/03/2020	477	0.123	3	0.176	186	0.222	3	0.227
26/03/2020	578	0.192	4	0.180	239	0.251	4	0.233
27/03/2020	759	0.272	5	0.199	304	0.241	5	0.235
28/03/2020	1019	0.295	6	0.215	403	0.282	6	0.243
29/03/2020	1228	0.187	7	0.211	490	0.195	7	0.236
30/03/2020	1408	0.137	8	0.201	560	0.134	8	0.223
31/03/2020	1789	0.239	9	0.206	682	0.197	9	0.220
01/04/2020	2352	0.274	10	0.212	858	0.230	10	0.221
02/04/2020	2921	0.217	11	0.213	1099	0.248	11	0.224
03/04/2020	3605	0.210	12	0.213	1230	0.113	12	0.214

*Taken from the Daily Telegraph.

Thus it is seen that the infection rate constant k is around 0.21, and applies to both the British and German data sets.

Hence k = <u>0.213</u> $days^{-1}$ From above table, equation 11, British data-set.

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Calculation of morbidity.

The mortality of the virus is the fraction of infected people who die. This fraction is given by equation 7:
Equation 7 is derived on sheet 5.

$$x_D = \frac{N_D(t)}{e^{k\cdot(t-\tau_D)}} \qquad \text{Equation 7}$$

Where:

$N_D(t)$ = Number of people killed by coronavirus at time t = t.
x_D = Mortality fraction i.e. fraction of infected people who are killed by the virus.
τ_D = Time to death in days from time of infection.

Data:

The observation of N_D is made on 3rd April 2020:

$N_D(t)$ = 3605

The pandemic in Britain is deemed to have started on 14th January 2020, based on reports in the press of British people returning from skiing holidays at this time and presenting with typical Coronavirus symptoms. This predates the earliest official British case by just over two weeks.

Start of infection = 14/01/2020
Date of observation = 03/04/2020
Hence time t = 80 days

The time to death from infection is deemed to be two weeks (14 days) as this is the characteristic duration of the illness (World Health Org.).

Hence τ_D = 14 days

The infection rate constant k is calculated on the foregoing sheet:

Hence k = 0.213 days^{-1}

From the above data the mortality is calculated from equation 7:

x_D = 0.0029 deaths/infections

The mortality may be expressed as a percenatage:

Percentage morbidity = 0.29 %

Calculation of the number of infected people on Friday 3rd April 2020.

The number of infections are given by equation 4 (see sheet 4) and characterised by the infection rate constant k.

$$N_I = e^{kt} \qquad \text{Equation 4}$$

Where:

t = Time in days.
N_I = Number of infected people at time t.
k = Infection rate constant.

Data:

t = 80 days
k = 0.213 days^{-1}
N_I = 24433911 People
N_I = 24.4 Millions of People

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Derivation of the equation of the number of Coronavirus cases.

The rate of progression of the coronavirus is given by the following ordinary differential equation:

$$\frac{dN_I}{dt} = k \cdot N_I \qquad \text{Equation 1}$$

Where:

t = Time in days.

N_I = Number of infected people at time t.

k = Infection rate constant.

dN_I/dt Rate of infection at time t.

Equation 1 is solved by separation of the variables and direct integration:

$$\int_1^{N_I} \frac{dN_I}{N_I} = \int_0^t k \cdot dt \qquad \text{Equation 2}$$

Note the integration limits of equation 2:

t = 0 $N_I = 1$ The pandemic begins with a single inefected person.

t = t $N_I = N_I$

The integration of equation 2 proceeds using the identity:

$$d(\ln f) = \frac{df}{f}$$

Where:

f = Any function that is differentiatable across the full set of the independent variable of interest i.e. t = 0 to t = t.

Whence equation 3 is obtained:

$$[\ln N_I]_1^{N_I} = [k \cdot t]_0^t$$

$$\ln N_I = k \cdot t \qquad \text{Equation 3}$$

Take inverse logarithms to base epislom on both sides to yield:

$$N_I = e^{kt} \qquad \text{Equation 4}$$

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Derivation of the equation for morbidity.

It is speculated that a constant fraction of people infected with corona virus will die, following a characteristic illness time.
Thus the number of deaths may be expressed by equation 5:

$$N_D(t) = x_D \cdot N_I(t - \tau_D)$$ Equation 5

Where:

$N_D(t)$ = Number of people killed by coronavirus at time t = t.
x_D = Mortality fraction i.e. fraction of infected people who are killed by the virus.
τ_D = Time to death in days from time of infection.
$N_I(t-\tau_D)$ = Number of people infected at time $t = t - \tau_D$.

In the foregoing sheet N_I was obtained as a function of time (equation 4):

$$N_I = e^{kt}$$ Equation 4

Substitute equation 4 into equation 5 to eliminate N_I:

$$N_D(t) = x_D \cdot e^{k \cdot (t - \tau_D)}$$ Equation 6

Re-arrange equation 6 to make the mortality fraction x_D the subject:

$$x_D = \frac{N_D(t)}{e^{k \cdot (t - \tau_D)}}$$ Equation 7

Observation of the deaths from the virus to calculate the infection rate constant k.

From equation 6 the number of deaths from the virus at t = t and t = t + Δt are given by

$$N_D(t) = x_D \cdot e^{k \cdot (t - \tau_D)}$$ Equation 6

$$N_D(t + \Delta t) = x_D \cdot e^{k \cdot (t + \Delta t - \tau_D)}$$ Equation 8

Where:
Δt = The time in days between successive observations of the total number of deaths due to the virus.
$N_D(t+\Delta t)$ = The number of deaths due to the virus at time $t = t + \Delta t$.

Divide equation 8 by equation 6:

$$\frac{N_D(t + \Delta t)}{N_D(t)} = e^{k \cdot \Delta t}$$ Equation 9

Take natural logarithms (i.e. to base epsilom) on both sides of equation 10, and re-arrange to make k the subject:

$$k = \frac{1}{\Delta t} \cdot \ln\left(\frac{N_D(t + \Delta t)}{N_D(t)}\right)$$ Equation 10

For observations taken on successive days $\Delta t = 1$ and equation 10 simplifies to:

$$k = \ln\left(\frac{N_D(t + 1)}{N_D(t)}\right)$$ Equation 11